# INVARIANT REGION PRESERVING SCHEMES FOR MIXED HYPERBOLIC-PARABOLIC SPDES


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## Abstract

Mixed hyperbolic-parabolic SPDEs encapsulate a large spectrum of equations that have broad applications in various fields. When the diffusion matrix is positive semi-definite, the resulting SPDE can experience strong degeneracy, inducing non-uniqueness and shock waves. To ensure uniqueness of weak solutions, appropriate entropy conditions that allow for discontinuous solutions are required. In addition, one has to account for stochastic perturbations. It is then a nontrivial task to design numerical schemes that preserve pointwise invariant regions, where the entropy solution remains inside the same domain as the initial condition. In this talk we showcase two invariant region preserving numerical schemes that relies on an operator-splitting of the deterministic and stochastic part of the equation

$$du + \left( \sum_{i=1}^{d} \partial_{x_i} f_i(u) - \sum_{i,j=1}^{d} \partial_{x_i} \big( a_{ij}(u) \partial_{x_j} u \big) \right) dt = \sigma(u)\, dB,$$

where $a$ is a symmetric and positive semi-definite matrix, $f$ is a non-linear flux, and $\sigma(u)dB$ is a cylindrical Wiener noise. The first scheme discretizes the deterministic part with a local discontinuous Galerkin method, while the stochastic part is discretized using a Lamperti transformation combined with an explicit Runge–Kutta method. Meanwhile, for the second scheme, the deterministic part is discretized by a finite difference scheme with cross derivative correctors, while the stochastic part is discretized with an artificial barrier method combined with the Euler–Maruyama scheme. The splitting approach is justified by theoretical results that shows that semi-discrete splitting solutions converge to the unique entropy solution of more general stochastic convection-diffusion equations. Moreover, numerical experiments confirm that the numerical schemes are stable and preserve both positivity and invariant regions.