

Bifurcations of Limit Cycles in DDEs: Theory & Software

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Abstract

Until now, no robust and efficient (numerical) method has been available to study bifurcations of limit cycles in delay differential equations (DDEs). In this talk, I will present explicit computational formulas for the critical normal form coefficients of all codimension 1 bifurcations of limit cycles (fold, period-doubling and Neimark-Sacker) in DDEs. These formulas are essential for detecting codimension 2 points and distinguishing between sub- and supercritical bifurcations.

The approach consists of four steps. First, we prove the existence of a smooth periodic finite-dimensional center manifold near a nonhyperbolic cycle [1]. Second, we show that the dynamics near such a cycle on the periodic center manifold can be described in terms of so-called periodic normal forms [2]. Third, we derive explicit formulas for the critical normal form coefficients in terms of the periodic (adjoint) (generalized) eigenfunctions of the characteristic operator, a generalization of the well-known characteristic matrix [3]. Finally, we illustrate the effectiveness of the method with numerical examples [4].

References

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