

Thermodynamical Allen–Cahn Equation with General Polynomial Free Energy

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Abstract

The Allen–Cahn equation was originally introduced to model of antiphase domain coarsening in a binary mixture in materials science. It is applied to various scientific fields such as data classification, volume reconstruction, image segmentation, multiphase flow, and biological transport networks, and dendritic crystal growth. To describe more natural and complex dynamics, we consider the thermodynamical conductivity into the Allen–Cahn equation with the general polynomial free energy. The conductivities are chosen based on the Fick’s law, Werde’s law, and Chapman’s law, which are based on the thermodynamics. Some properties of the governing equation are analyzed based on the modified Ginzburg–Landau energy. To numerically solve the proposed governing equation, we construct a numerical scheme based on the linear stabilized splitting scheme which treats the nonlinear term explicitly and the linear term as implicitly. It is an efficient, unconditionally energy-stable, and property-preserving scheme. We numerically analyze and simulate some properties of the proposed numerical scheme in one-, two-, and three-dimensional spaces.

References

- [1] Allen, Samuel M and Cahn, John W, "A microscopic theory for antiphase boundary motion and its application to antiphase domain coarsening", *Acta metallurgica*, Vol. 27, No. 6, 1979, pp. 1085–1095
- [2] Kim, Yong-Jung and Seo, Hyowon, "Model for heterogeneous diffusion", *SIAM Journal on Applied Mathematics*, Vol 81, No. 2, 2021, pp. 335–354