

Initial Layer for the Boltzmann Equation

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Abstract

In studying the relation between the Boltzmann equation in kinetic theory and fluid dynamics, it is crucial to understand the singular layers that appear in the Boltzmann solutions. These include the boundary, shock, and initial layers, which become singular when the mean free path is small. The present work focuses on the initial layer, whose main feature is the propagation of discontinuities in the solutions of the Boltzmann equation. To analyze the global structure of Boltzmann solutions, it is essential to describe these discontinuities precisely in both the space-time variables (x, t) and the microscopic velocity ξ .

We construct the explicit form of the Green's function and use it to represent the Boltzmann solution. The quantitative structure of this Green's function enables a detailed analysis of the commutator between the nonlinear collision operator and the transport operator, providing a precise understanding of the propagation of discontinuities. Our results reveal both the explicit form and kinetic nature of these discontinuities, as well as the emergence of fluid-like waves in the initial layer. This explicit construction bridges the Boltzmann equation and hydrodynamic behavior. This work is in collaboration with Tai-Ping Liu and Shih-Hsien Yu.