

Existence of Solutions to the Navier-Stokes Equations in Lorentz-Besov Spaces

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Abstract

In this paper, we investigate the existence of local mild solutions to the three-dimensional incompressible Navier-Stokes equations in the framework of Lorentz-Besov spaces. The external force is assumed to belong to the Lorentz space $L^{1,q}(0, T)$, $T > 0$. We first establish maximal parabolic regularity in the Lorentz space $L^{1,q}(0, T; \dot{B}_{p,1}^0)$, where $1 < p < \infty$ and $1 < q \leq \infty$. Using this regularity result, we prove the existence of a local mild solution. The obtained solution lies in the Lorentz space $L^{\alpha,q}(0, T)$, $1 < \alpha < \infty$ with respect to time and in the homogeneous Besov space $\dot{B}_{p,1}^0$ with respect to space. The principal difficulty arises from a time singularity appearing in the estimates within the Lorentz space $L^{1,q}(0, T)$.