Second Positive Solution for Spatially Nonhomogeneous Elliptic Problem

Lukáš Kotrla

University of West Bohemia in Pilsen, Czech Republic
kotrla@kma.zcu.cz

Abstract

We consider the semilinear Dirichlet problem

$$\begin{cases} -\Delta u(x) &= \lambda\,|u(x)|^{q(x)-2}\,u(x)\,, & x \in \Omega\,, \\ u(x) &= 0\,, & x \in \partial\Omega\,. \end{cases}$$

Here, $\Omega \subset \mathbb{R}^N$ is a bounded domain with a sufficiently smooth boundary $\partial\Omega$ and $\lambda \geq 0$ is a spectral parameter. The variable exponent $q(x)$ is continuous and satisfies $1 < q^- \leq q(x) \leq q^+$. Specifically, we assume $q^- \leq q(x) < 2$ and $2 < q(x) \leq q^+$ on nonempty open subsets Ω_- and Ω_+ of Ω, respectively. Our attention is paid to the existence and multiplicity of positive weak solutions depending on the value of λ. The interplay between the convex and concave parts of the reaction term leads to the existence of thresholds $\lambda_0 > 0$ and $\lambda^* > 0$ such that a C^1-solution exists for any $\lambda \in (\lambda_0, \lambda^*)$. In addition, a second C^1-solution exists for $\lambda \in [a, b]$, where $\lambda_0 < a < b < \lambda^*$.

The talk extends the presentations of my co-authors Jiří Benedikt, Petr Girg, and Peter Takáč in minisymposium P11. This talk is focused particularly on the existence of a second solution, proven via Leray-Schauder degree theory.

References

[1] Benedikt, J., Girg, P., Kotrla, L., and Takáč, P. *On the Existence of Two Positive Solutions for Spatially Nonhomogeneous Elliptic Problems.* Mathematica Slovaca, to appear.