

# On the Mean Curvature Flow of Closed Curves Evolving on Surfaces and Two Dimensional Manifolds

Miroslav Kolář

Czech Technical University in Prague, Czech Republic  
MIROSLAV.KOLAR@FJFI.CVUT.CZ

## Abstract

We present an overview of our investigation of the flow of a family of closed curves evolving in the normal direction according to a prescribed geometric evolution equation either on a given two dimensional surface or on a two dimensional manifold which is embedded or immersed in the three-dimensional Euclidean space.

In the case of evolution on a surface given as a graph of two-dimensional function, we study two non-local geometric flows depending on the geodesic curvature and preserving either the total length of the surface curve, or its enclosed surface area. We treat these evolution equations by the method of vertical projection to the underlying plane. For such a projected flow, the particular planar normal velocity and external force are constructed. For a motion on two-dimensional manifolds, we derive a system of nonlinear parabolic equations that describe their motion and a possible attraction to the manifold.

Both considered geometric flows are treated by the parametric method, and we discuss some fundamental properties of such flows. The resulting evolution equations for parametrizations of those surface curves are solved numerically by means of the flowing finite volume method, which is modified by the suitable choice of the tangential velocity ensuring appropriate redistribution of the discretization points, which helps to stabilize the numerical algorithm. Qualitative and quantitative computational results demonstrating the spatio-temporal behavior of the solution, and the performance of the numerical algorithm, are included.

Our presented results are based on our papers [1, 2, 3, 4].

## References

- [1] M. Kolář and D. Ševčovič, *Mean Curvature Flow of Closed Curves Evolving in Two Dimensional Manifolds*, Japan Journal of Industrial and Applied Mathematics, 43(16), (2026).
- [2] M. Kolář, M. Beneš and D. Ševčovič, *Qualitative and numerical aspects of a motion of a family of interacting curves*, SIAM Journal on Applied Mathematics, 82(2), (2022), 549-575.
- [3] M. Kolář, M. Beneš and D. Ševčovič, *On Surface Area and Length Preserving Flows of Closed Curves on a Given Surface*. In: Numerical Mathematics and Advanced Applications ENUMATH 2017, (2019).
- [4] M. Kolář, M. Beneš and D. Ševčovič, *Area preserving geodesic curvature driven flow of closed curves on a surface*, Discrete and Continuous Dynamical Systems - B, 22 (2017), 3671–3689.