A Numerical Algorithm for Solving Singularly Perturbed Boundary Value Problem Arising in Neuronal Variability Model

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Abstract

In this research, we present a robust computational approach that effectively employs an exponentially fitted mesh cubic spline for the numerical treatment of a mathematical model associated with neuronal variability. This model decisively determines the expected time required for generating an action potential in nerve cells in response to random synaptic inputs into dendrites, framed as a general boundary value problem for a singularly perturbed differential-difference equation with small shifts. We construct a highly effective numerical scheme and establish a parameter-uniform error estimate that clearly demonstrates its reliability. Through a series of test examples, we thoroughly investigate the performance of this scheme, specifically analyzing how the size of the delay argument and the coefficient of the delay term influence the solution's layer behavior. The results obtained unequivocally align with the theoretical predictions, validating the efficacy of our approach.

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