

Unconditionally Stable Numerical Solution for the High-Order Allen–Cahn Equation

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Abstract

We present an unconditionally stable algorithm for the Allen–Cahn (AC) equation incorporating a high-order free energy formulation. This high-order AC model enhances the preservation of interfacial dynamics and effectively suppresses noise. The proposed scheme guarantees unconditional stability, which is crucial for accurate phase transition modeling and maintaining fine structural details. To solve the governing equation efficiently, it is split into two subproblems that are solved independently. The nonlinear term is treated using a frozen coefficient approach, followed by an analytical solution. The linear term is solved using the discrete cosine transform. To demonstrate the effectiveness of the method, we perform several numerical simulations in both two- and three-dimensional domains. Thanks to its unconditional stability, the algorithm allows large time steps without compromising stability. Furthermore, we explore the distinct features of the high-order AC equation, particularly its improved capability to model phase separation under strong noise and intricate interface structures.

References

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