

Distributional Adjoint Theory for Retarded Differential Operators: Uncovering the Structural Origin of Hale's Bilinear Form

Junya Nishiguchi

Setsunan University, Osaka, Japan / Tohoku University, Miyagi, Japan
JUNYA.NISHIGUCHI@SETSUNAN.AC.JP / JUNYA.NISHIGUCHI.B1@TOHOKU.AC.JP

Abstract

The bilinear form introduced by Hale plays a pivotal role in the adjoint theory and spectral analysis of retarded functional differential equations (RFDEs). However, in the case of systems of RFDEs, this bilinear form is often presented as a prescribed definition, and its structural origin as a boundary term—especially regarding the rigorous ordering of matrix-valued measures—has not been fully elucidated.

In this talk, we provide a self-contained reconstruction of the linear RFDE theory from a distributional perspective. We characterize the fundamental matrix as a distributional matrix solution by considering an RFDE with a Dirac delta function as an external force. This result provides a rigorous formulation of the impulse response matrix for general retarded systems. In addition, we demonstrate that Hale's bilinear form naturally emerges as a boundary term in a Green-type identity. Our approach ensures the rigorous ordering of matrix-valued integrals and clarifies the essential role of constant extensions of functions at the boundaries. These results provide a solid mathematical foundation for the adjoint structures in systems of RFDEs.