

Sharp Oscillation Criteria for n th Order Linear Delay Differential Equations

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Abstract

We establish a sharp oscillation criterion for n -th order linear delay differential equations with positive continuous coefficients and general delay argument. The decisive oscillation constant is the maximum of an explicitly constructed exponential-polynomial, determined by the order and the asymptotic time-delay ratio. Its optimality is shown by exact agreement with an Euler-type proportional-delay differential equation, extending recent sharp results for lower orders: $n = 2$ (in a more general half-linear setting) by Jadlovská and Džurina [1], $n = 3$ by Graef, Jadlovská, and Tunç [2], and $n = 4$ by Jadlovská et al. [3]. If this constant exceeds the critical value, Property A holds: every solution oscillates for n even, while for odd n every solution either oscillates or tends to zero together with all derivatives up to $n - 1$. Beyond identifying this constant, we develop an iterative process, built from suitably chosen $\liminf$ constants, that yields two-sided comparison inequalities between arbitrary derivative orders of any nonoscillatory solution, in a precise binomial-coefficient form. We illustrate the scope of our results on broad classes of equations with asymptotically proportional, sublinear, or piecewise-proportional delays, as well as slowly varying or oscillatory coefficients. *This is a joint work with Agacik Zafer and Tongxing Li [4].*

References

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