

# Oscillation Theorems for Linear Differential Equations with a Proportional-Derivative Controller

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## Abstract

This study is based on joint work with Professor M. Onitsuka of Okayama University of Science (including [5]). Recently, considerable attention has been devoted to oscillation theory for linear differential equations with proportional-derivative (PD) control (see, e.g., [1, 2, 3, 4]); nevertheless, this theory remains in its early stages of development. In this work, we investigate the oscillatory and nonoscillatory behavior of solutions to these equations, with the aim of extending classical oscillation theory to this framework.

In particular, we establish a Leighton–Wintner-type oscillation theorem and Hille–Kneser-type oscillation and nonoscillation criteria in the presence of a PD controller. The Leighton–Wintner-type result is obtained by means of the Riccati technique, suitably adapted to the PD structure of the equation. Furthermore, by clarifying the oscillation constant for Euler-type equations with PD control, we derive Hille–Kneser-type criteria that characterize both oscillatory and nonoscillatory behavior. This work was supported in part by JSPS KAKENHI Grant No. JP22K13933 and by the Sumitomo Foundation Grant for Basic Science Research Projects, Grant No. 2402156.

## References

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