

Self-Similar Solutions with Finite Time Blow-up for Reaction-Diffusion Equations

Razvan Gabriel Iagar

Universidad Rey Juan Carlos, Madrid, Spain
RAZVAN.IAGAR@URJC.ES

Abstract

This talk focuses on the existence of self-similar solutions presenting finite time blow-up, in the form

$$u(x, t) = (T - t)^{-\alpha} u(|x|(T - t)^{-\beta}), \quad (x, t) \in \mathbb{R}^N \times (0, T), \quad (1)$$

for the classical quasilinear reaction-diffusion equation

$$\partial_t u = \Delta u^m + u^p, \quad p > m > 1. \quad (2)$$

For $m = 1$, Lepin [3] proposed the following critical exponent

$$p_L = \begin{cases} 1 + \frac{6}{N-10}, & \text{if } N \geq 11 \\ \infty, & \text{if } 1 \leq N \leq 10, \end{cases}$$

as the maximal value of p for which the reaction-diffusion equation admits solutions in the self-similar form (1). This fact has been proved by Mizoguchi in [4]. One natural question was whether there is a similar exponent with $m > 1$, and we give a negative answer to this question: we prove that, for any $m > 1$ and $p > m$ large, Eq. (2) admits self-similar solutions in the form (1), in strong contrast to the semilinear case. We export the same techniques to the Hardy-Hénon equation in $\mathbb{R}^N \times (0, \infty)$

$$\partial_t u = \Delta u + |x|^\sigma u^p, \quad p > 1, \quad \sigma > \max\{-2, -N\},$$

and show that no Lepin-type exponent exists for $\sigma \geq 2$, while we identify the expression of the critical Lepin-type exponent for $\sigma \in (-2, 2)$.

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References

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