

# Order Reduction of Exponential Runge–Kutta Methods: Non-Commuting Operators

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## Abstract

Nonlinear parabolic equations are central to numerous applications in science and engineering, posing significant challenges for analytical solutions and necessitating efficient numerical methods. Exponential integrators have recently gained attention for handling stiff differential equations. This paper explores exponential Runge–Kutta methods for solving such equations, focusing on the simplified form  $u'(t) + Au(t) = Bu(t)$ , where  $A$  generates an analytic semigroup and  $B$  is relatively bounded with respect to  $A$ . By treating  $A$  exactly and  $B$  explicitly, we derive error bounds for exponential Runge–Kutta methods up to third order. Our analysis shows that the first- and second-order methods maintain their order under mild regularity conditions on the initial data  $u_0$ , while also addressing the phenomenon of order reduction in third-order methods. Through careful convergence analysis and numerical investigations, this study provides a comprehensive understanding of the applicability and limitations of exponential Runge–Kutta methods in solving linear parabolic equations involving two unbounded and non-commuting operators.

## References

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