Robust Stability of a Class of Complex Balanced Chemical Reaction Networks with Time Delays and Time Varying Perturbations

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Abstract

Chemical reaction network theory (CRNT) provides methods for analyzing the dynamics of solutions to ordinary differential equations (ODEs) describing the time evolution of species concentrations in chemical reaction networks (CRNs) based on their topological structures. In particular, the Deficiency Zero Theorem provides a sufficient condition for the ODEs to possess a unique asymptotically stable equilibrium point, known as a complex balanced equilibrium [1]. Recently, CRN systems have been extended to incorporate production delays and environmental fluctuations in reactions. Consequently, the dynamics of such systems are described by non-autonomous delay differential equations (DDEs). While the asymptotic stability of an equilibrium for autonomous DDEs of complex balanced CRNs [2] and the persistence of non-autonomous DDEs for delayed CRNs with time-varying kinetics [3] have been proven, robust stability for systems with both delays and time-varying perturbations has not yet been established. In this talk, we consider a class of complex balanced and conservative CRNs satisfying the conditions of the DZT and provide a sufficient condition for the robust stability of non-autonomous DDEs subjected to both arbitrary production delays and time-varying perturbations in reaction rates. In order to prove this, we define a Lyapunov-Krasovskii functional for the non-autonomous DDEs, extending the one used for autonomous systems [2]. Under the assumption that the perturbations are bounded within a specific range, we evaluate the time derivative along the solutions and prove that the solutions to the DDEs are globally ultimately bounded. Moreover, we also show that the solutions to the DDEs converge to an equilibrium point when the perturbations vanish as time goes to infinity.

References

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