Periodic Solutions of Cosmology Inspired Hamiltonian System

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Abstract

This is a joint work with prof. Sławomir Rybicki.
In this talk, we investigate the existence and structure of sets of nonstationary periodic solutions of a Hamiltonian system motivated by cosmology. The system is given by

$$\dot{u}(t) = JH'(u(t)), \tag{1}$$

where $u = (x, y, z, X, Y, Z)$ and the Hamiltonian H is defined as

$$H(x, y, z, X, Y, Z) = \tag{2}$$

$$= \frac{1}{2}\left(x^2 + X^2 - y^2 - Y^2 + z^2 + Z^2\right) +$$

$$+\epsilon^2\left(\frac{\alpha}{4}\left(x^4 + y^4 + z^4\right) + \frac{\beta}{2}\left(x^2y^2 + x^2z^2 + y^2z^2\right)\right).$$

We use two approaches to study sets of nonstationary periodic solutions of system (1) with Hamiltonian (2). The first is based on bifurcation theory, while the second relies on equivariant topology and the representation theory of compact Lie groups. First, we introduce the parametrized system $\dot{u}(t) = \lambda JH'(u(t))$ and search for 2π-periodic solutions, which correspond to $2\pi\lambda$-periodic solutions of the original system (1). We prove the existence of global bifurcation points of nonstationary 2π-periodic solutions of parametrized Hamiltonian system and describe how their number depends on the parameters α and β. Furthermore, we provide lower bounds on the number of unbounded continua of nonstationary 2π-periodic solutions bifurcating from stationary ones. In addition, we study nonstationary periodic solutions of system (1) that bifurcate from nonisolated stationary solutions. Exploiting the S^1-invariance of the Hamiltonian (2), we establish the existence of connected families of nonstationary periodic solutions bifurcating from the manifold of stationary solutions.