

Chaos in the Hyperion Tumbling Model

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Abstract

In a joint work with Amadeu Delshams (UPC Barcelona) and Piotr Zgliczyński (JU) we study the three-axis inner rotation (*tumbling*) of an ellipsoidal satellite in a Keplerian orbit. In our previous paper [1] we considered a simplified uniaxial case, with the axis of rotation perpendicular to the orbital plane [2]. The system of three ordinary differential equations is periodic with respect to one of the variables (*true anomaly* f), which allowed us to study the Poincaré section $f = 0 = 2\pi$. The system exhibited chaotic behavior for some parameters (the shape of the ellipsoid and the orbital eccentricity e). We were particularly interested in the values corresponding to Hyperion, one of Saturn's moons, known for its non-spherical shape and apparently chaotic inner rotation.

The uniaxial model does not provide a full physical description of Hyperion's tumbling, which is clearly triaxial. The full model, based on Euler's rigid-body equations, leads to a much more complicated system of seven ordinary differential equations (omitted here for reasons of size; see [3]), of which the simplified uniaxial model is an invariant subspace. For this reason, it too is chaotic, but we expect the structure of this chaos to be more complex in the 7-dimensional space: perhaps it is divided into chaotic regions connected by homo- and heteroclinic orbits.

Using the Interval Newton Method implemented in C++ with the use of CAPD library [4] for computer-assisted proofs, we found a number of periodic orbits for the analogous Poincaré map in the full space. The current work is to study the behavior of their stable and unstable manifolds which seem to intersect transversely. This, in turn, we expect to prove rigorously with CAPD. The poster presents the up-to date numerical results and plans for future work.

References

- [1] Anna Gierzkiewicz and Piotr Zgliczyński. *A computer-assisted proof of symbolic dynamics in Hyperion's rotation*. Celestial Mechanics and Dynamical Astronomy, 131(7):33, 2019.
- [2] J. Wisdom, S. J. Peale, and F. Mignard. *The chaotic rotation of Hyperion*. Icarus, 58(2):137 – 152, 1984.
- [3] James Jay Klavetter. *Rotation of Hyperion. II – Dynamics*. Astronomical Journal, 98:1855–1874, 11 1989.
- [4] T. Kapela, M. Mrozek, D. Wilczak and P. Zgliczyński. *CAPD::DynSys: a flexible C++ toolbox for rigorous numerical analysis of dynamical systems*. Communications in Nonlinear Science and Numerical Simulation, 101:105578, 2021.