Large Time-Periodic Solutions to Nonlinear Wave Equations

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Abstract

We study time-periodic solutions for nonlinear wave equations on an interval with Dirichlet boundary conditions and find intricate bifurcation structures forming fractal-like patterns [1, 2]. In addition to the small solutions with frequencies belonging to nowhere dense sets known from the classic works, see for example [3, 4], they include a new class of large-energy solutions with complex mode compositions. We perform the first proof of existence of solutions belonging to this class [5]. Similar phenomena can be observed for other systems [6, 7]. This is a joint work with Maciej Maliborski.

References

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