A Parabolic-Elliptic-Hyperbolic System of Keller-Segel Type: Numerical Analysis and Simulation

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Abstract

This talk aims to present a numerical study of a system of partial differential equations consisting of a parabolic, an elliptic, and a hyperbolic equation, which can be used to describe cell migration in elastic tissues by integrating chemotaxis, interstitial fluid pressure, and tissue displacement.

The system couples a Keller–Segel-type model for cell density and chemical concentration with an elliptic equation for fluid pressure and a wave-type equation for tissue displacement. The convective cell velocity depends nonlinearly on the chemoattractant gradient, Darcy's velocity induced by fluid pressure, and the velocity of tissue displacement, whereas the convective velocity of the chemoattractant depends only on Darcy's velocity.

We propose a semidiscrete nonuniform finite difference method to approximate the continuous system, which can be interpreted as a fully discrete-in-space, piecewise linear finite element method.

Furthermore, we derive second-order spatial error estimates for all system variables using a suitable discrete norm. Finally, numerical simulations illustrate the theoretical results and demonstrate the interplay between chemotaxis and tissue mechanics.

We remark that the present work improves upon the results presented in [2], where we studied, from a numerical perspective, a Keller–Segel–flow model describing cell migration driven by the spatial distribution of a chemoattractant enhanced by interstitial fluid pressure. For a review of several extensions of the classical Keller–Segel model for cell migration, we refer the reader to [1].

References

[1] Gurusamy Arumugam, Jagmohan Tyagi. Keller-Segel chemotaxis models: a review. *Acta Applicandae Mathematicae*, 171 (1):6, 2021.

[2] Augusto Fernandes, José Augusto Ferreira, Luís Pinto. Numerical Analysis of a Keller-Segel-Flow model for tumor cell migration. *Mathematics and Computers in Simulation*, 238: 65–81, 2025.