Multiplicity of solutions for nonlinear differential systems using fixed point theory in product spaces

Laura Mª Fernández–Pardo

Universidade de Santiago de Compostela, Santiago de Compostela, Spain.
LAURA.FERNANDEZ.PARDO@USC.ES

Abstract

In this talk we present sufficient conditions for the existence and multiplicity of solutions to nonlinear differential systems using fixed point theory in cones. To establish these conditions, the problem is reformulated as an equivalent fixed point problem for compact operators of the form $T = (T_1, T_2)$ acting on a product space $X \times Y$.

In recent works [1, 3], component-wise Krasnosel'skii-type fixed point conditions have been imposed on operators of this form. In this direction, we show that these conditions can be combined with classical multiplicity results, such as the Leggett–Williams theorem [2], yielding multiplicity results in product spaces. Many of the fixed points obtained by these methods have the important property that both components are positive. Therefore, when applied to nonlinear differential systems, these results yield multiple solutions with all components nontrivial.

References

[1] L. M. Fernández-Pardo and J. Rodríguez-López, Component-wise Krasnosel'skii type fixed point theorem in product spaces and applications, *Nonlinear Anal. Real World Appl.*, **88** (2026), No. 104506, 1-12.

[2] R. W. Leggett and L. R. Williams, Multiple positive fixed points of nonlinear operators on ordered Banach spaces, *Indiana Univ. Math. J.*, **28** 4 (1979), 673-688.

[3] J. Rodríguez-López, A fixed point index approach to Krasnosel'skiĭ-Precup fixed point theorem in cones and applications, *Nonlinear Anal.*, **226** (2023), No. 113138, 1–19.