The Vortex Filament Equation in the Energy-Variational Framework

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Abstract

Strongly localized vortical structures are frequently observed in fluid dynamics. A classical model for the motion of vortex filaments is the equation for binormal curvature flow. However, this differential equation for the parametrization of a closed, oriented curve fails to capture the formation of corners and kinks, as well as the finite-time blow-up of smooth solutions. This limitation motivates a relaxation of the solution concept. We begin by considering a weak formulation, which naturally extends to divergence-free measures instead of curves. In this framework, the length of the curve, which is an invariant preserved by smooth solutions and serving as an energy, is replaced with the total variation of the measure. As global existence of weak solutions cannot be expected, we introduce energy-variational solutions, a concept that augments the combination of the energy inequality and the weak formulation with an additional defect term. A general result for abstract evolution equations enables to derive the global-in-time existence of energy-variational solutions in this framework. Additionally, we discuss a weak-strong uniqueness principle that guarantees that energy-variational solutions coincide with classical solutions as long as the latter exist.

References

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