**Navier–Stokes–Cahn–Hilliard System in a 3D Perforated Domain with Free Slip and Source Term: Existence and Homogenization**

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**Abstract**

We study a diffuse–interface model for a binary incompressible mixture in a periodically perforated porous medium, described by a time–dependent Navier–Stokes–Cahn–Hilliard (NSCH) system posed on the pore domain $\Omega_p^\varepsilon \subset \mathbb{R}^3$. The microscopic model involves a variable viscosity tensor, a non–conservative source term in the Cahn–Hilliard equation, and mixed boundary conditions consisting of no–slip on the exterior boundary and Navier slip with zero tangential stress on the surfaces of the solid inclusions. The capillarity strength $\lambda^\varepsilon > 0$ depends on the microscopic scale $\varepsilon > 0$.

For each fixed $\varepsilon > 0$, we establish the existence of weak solutions on a finite time interval $(0, T)$ together with a priori estimates that are uniform with respect to $\varepsilon$ (and $\lambda^\varepsilon$). A major analytical difficulty stems from the presence of the source term, which breaks mass conservation and prevents a direct application of the classical Poincaré–Wirtinger inequality. This issue is resolved through refined estimates for the spatial mean combined with suitable extension and restriction operators adapted to the perforated geometry.

We then investigate the asymptotic behavior of the system as $\varepsilon \to 0$ and perform a rigorous periodic homogenization using the periodic unfolding method. The uniform estimates provide the compactness required to identify the macroscopic limit fields and to pass to the limit in the nonlinear coupling terms, yielding effective tensors that encode the influence of the microscopic geometry. Depending on the limit value $\lambda = \lim_{\varepsilon\to 0} \lambda^\varepsilon \in [0, +\infty)$, two qualitatively different homogenized models emerge. In the vanishing capillarity regime $\lambda = 0$, the limit system is of Stokes–Cahn–Hilliard type and exhibits no macroscopic convection or phase advection, corresponding to creeping–flow behavior. In contrast, when $\lambda \in (0, +\infty)$, the homogenized equations retain nonlinear convection and advective transport, leading to a genuine Navier–Stokes–Cahn–Hilliard structure at the macroscopic scale.

Finally, we prove the convergence of the microscopic free energy towards a homogenized energy functional satisfying a corresponding dissipation law.

# References

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