Time Filtered Finite Difference Schemes for Hyperbolic Problems

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Abstract

The focus of this presentation is to investigate the effects of combining a time filtering scheme with various finite difference methods for partial differential equations in conservation law form. Time filters have been used for more than fifty years, with the first version introduced by Robert in 1966 [6], and subsequently analyzed and expanded by many scholars [1, 2, 4, 5, 7, 9]. The work presented here is motivated by a time filter proposed by Guzel and Layton in [3]. We show that the filter can be combined with three classical algorithms when applied to the prototype linear hyperbolic equation to result in filtered methods that are modular and require minimal modification (adding only one line of code) to achieve increased accuracy without increased computational expense. When assessing the stability and convergence properties of the filtered versions of the schemes, the examples demonstrate different possible outcomes when one attempts to apply filtering. In some cases, the filter technique can be successfully combined with an explicit scheme to yield one that is more accurate than its unfiltered counterpart. Von Neumann analysis [8] proves useful for determining the stability properties of the filtered versions of the upwind and leapfrog methods as it allows one to deal with the filter parameter systematically. With a careful choice of this parameter, the filtered upwind scheme is shown to be more accurate than its upwind counterpart, revealing that a new CFL condition is required for stability.

Our analysis demonstrates that the type of filter considered here cannot remedy certain kinds of stability properties. The filtered leapfrog scheme is shown to inherit the same type of spurious mode that its original unfiltered counterpart is known to possess. We show that combining the filter with the Crank-Nicolson method leads to an implicit method where no value of the filter parameter provides for a consistent filtered version of the method. While the analysis of the aforementioned methods is carried out for the linear hyperbolic equation, we also demonstrate that the filtering scheme can be combined with both fully explicit and semi-implicit treatments of an approximation scheme for the nonlinear Lighthill-Whitham-Richards model. The results for this case show that the improvements gained through filtering are less pronounced. Simulation results are given for both a shock wave example and a rarefaction wave case. The presentation includes a survey of computational results that support the theoretical results for the improved accuracy of the filtered upwind scheme along with examples of the time filter's limited effectiveness for the nonlinear model case.

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