The Least-Squares Method in the Theory of Boundary Value Problems with Delay

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Abstract

The present work addresses the issue of solution existence

$$z(t,\varepsilon): \ z(\cdot,\varepsilon) \in \mathbb{C}^1[0,T], \ z(t,\cdot) \in \mathbb{C}[0,\varepsilon_0]$$

for nonlinear periodic boundary value problem with delay [1]

$$dz(t,\varepsilon)/dt = A(t)z(t,\varepsilon) + B(t)z(t-\Delta,\varepsilon) + f(t) + \\ +\varepsilon\, Z(z(t,\varepsilon), z(t-\Delta,\varepsilon), t, \varepsilon) \qquad (1)$$

in a small neighborhood of the solution $z_0(t) \in \mathbb{C}^1[0,T]$ of the generating problem

$$dz_0(t)/dt = A(t)z_0(t) + B(t)z_0(t-\Delta) + f(t), \quad \Delta \in \mathbb{R}^1. \qquad (2)$$

Here, $A(t), B(t)$ are $(n \times n)$ dimensional matrices, $Z(z(t,\varepsilon), z(t-\Delta,\varepsilon), t, \varepsilon)$ is a nonlinear vector function, T-periodic to the independent variable t, analytic with respect to the unknown $z(t,\varepsilon)$ and $z(t-\Delta,\varepsilon)$ in a small neighborhood of the solution of the generating problem (2) and continuous with respect to the small parameter ε on the interval $[0,\varepsilon_0]$. In addition, the function $f(t)$ is continuous with respect to the $t \in [0,T]$.

By applying the Adomian decomposition method and the least squares method scheme, we have derived the necessary and sufficient conditions for the existence of solutions of the weakly nonlinear periodic boundary value problem for a system of differential equations with concentrated delay in the critical case [3]. The efficiency of the iterative schemes we have developed is demonstrated using an example of solving the problem of approximating periodic solutions to an equation with concentrated delay, which models a non-isothermal chemical reaction [1, 2].

References

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