Analysis of Anisotropic and Heterogeneous Models with Applications to Complex Materials

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Abstract

We focus on developing new mathematical methods for the analysis of nonlinear processes in multiphase structures, modeled by elliptic and parabolic partial differential equations with anisotropic and heterogeneous behavior. These models describe complex phenomena in materials such as multiphase solid and liquid materials, porous media, semiconductor devices like Organic Light-Emitting Diodes (OLEDs), etc. Main attention will be given to the following models: evolution anisotropic porous medium equation:

$$u_t = \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \left(|u|^{m_i - 1} u_{x_i} \right) + f,$$

$$1 - \frac{2}{n} < m_1 \leq m_2 \leq \cdots \leq m_n < \bar{m} + \frac{2}{n},\ \bar{m} = \frac{1}{n} \sum_{i=1}^{n} m_i,\ f \in L^1(\Omega_T),\ \Omega_T = \Omega \times (0,\, T),$$

and anisotropic $p-$ Laplace equation:

$$\sum_{i=1}^{n} \frac{\partial}{\partial x_i} \left(\left(|\nabla u|^{p_i - 2} \right) u_{x_i} \right) = f \in L^1(\Omega),$$

$$\frac{2n}{n+1} < p_1 \leq p_2 \leq \cdots \leq p_n < 2 + \frac{k}{n},\ k = n(\bar{p} - 2) + \bar{p},\ \bar{p} = \frac{1}{n} \sum_{i=1}^{n} p_i.$$

We develop an approach ([1]) that eliminates the need to handle degenerate and singular cases separately. This approach provides potential estimates for solutions of anisotropic equations and, as a result, proves the regularity result (local continuity) of weak solutions for all cases.

References

[1] K. Buryachenko, I. Skrypnik I, Local Continuity and Harnack's Inequality for Double-Phase Parabolic Equations.*Potential Anal* **56** (2022), no (1), 137–164.