Fourth-Order Diffusion Equations and Rényi Entropies

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Abstract

In this talk we discuss the asymptotic behavior of strong solutions to a family of nonlinear fourth-order diffusion equations [1, 5]

$$\partial_t u = -\frac{1}{p-1}\left(u^p\left(u^{p-1}-1\right)_{xx}\right)_{xx}, \quad x \in \mathbb{R},\ t>0,$$

with particular focus on the thin-film equation

$$\partial_t u = -(uu_{xxx})_x,$$

which is obtained for $p = 3/2$. The method builds on the framework introduced by Carrillo and Toscani [4] for second-order diffusion equations — by introducing a time-dependent rescaling that preserves the second moment, we establish sharp convergence rates toward the steady state in terms of the relative Rényi entropy. Compared to rates derived from the dissipation of the classical relative entropy [3, 5], this approach yields improved estimates at early and intermediate times, and consequently a sharper convergence in the L^1-norm [2]. The method formally applies to the family of fourth-order equations, including the well-known Derrida-Lebowitz-Speer-Spohn (DLSS) equation ($p = 1$), and can be rigorously justified for strong solutions of the thin-film equation.

References

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