

Graph-Based Semi-Supervised Learning via PDEs and Total Variation

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Abstract

Graph-based semi-supervised learning seeks to propagate class labels from a small labeled set to a large unlabeled graph by exploiting its intrinsic geometry. This talk surveys a line of work developing increasingly refined PDE-theoretic frameworks for this problem [1, 4].

We begin with segregation-based classification [1], where a quadratic Dirichlet energy is minimized subject to pairwise disjointness constraints motivated by spatially competitive reaction–diffusion systems. The resulting model admits a unique minimizer whose fixed-point characterization yields a simple convergent algorithm. We then address class imbalance and convergence issues arising in Laplace and Poisson learning through modified random-walk schemes [2], and extend the segregation paradigm to the infinity Laplacian setting connecting label propagation to Lipschitz extensions on graphs.

The central contribution is Total Variation Segregated Learning (TVSL) [4], which replaces the quadratic energy with graph total variation. This yields piecewise-constant label functions with sharp decision boundaries, an exact min-cut equivalence in the binary case, and a tractable proximal-gradient algorithm with PDHG inner solvers enforcing competition at each iteration. Convergence guarantees and numerical experiments on benchmark datasets are presented.

References

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