

Energy Consistent Hyperbolic Approximations for Some Higher Order PDEs

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Abstract

We introduce novel energy-consistent hyperbolic relaxation models for approximating the solutions of third-order and fourth-order nonlinear PDEs. In particular, we first propose a hyperbolic and hyperbolic-parabolic system for a class of diffusive-dispersive PDEs using a novel relaxation approach. We derive an explicit energy structure of the proposed system, which converges to the energy (Lyapunov) functional of the original equation when relaxation parameters vanish. This, in particular, demonstrates that the relaxation system preserves the key structural properties of the underlying model. Relying on the relative entropy method, we prove the convergence of solutions of these approximate systems to the solutions of the diffusive-dispersive equations [2]. We then show the scope of this approach to a highly complex fourth-order thin-film type equation and propose a novel first-order system approximating the solutions of the fourth-order thin-film/ Cahn-Hilliard type equation in arbitrary space dimensions. Again, by utilizing the relative entropy framework, we succeed in proving the convergence of the weak entropy solutions of the first-order system to the sufficiently smooth solution of the original equation [1]. Some test cases for a variety of physical PDEs are provided in the end to validate the analysis and the scope of our approach.

This is a joint work with **Christian Rohde** (University of Stuttgart, Stuttgart, Germany) and **Firas Dhaouadi** (INRIA Bordeaux, France).

References

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- [2] R. Barthwal, F. Dhaouadi, and C. Rohde. On hyperbolic approximations for a class of dispersive and diffusive-dispersive equations. *arXiv preprint arXiv:2512.04882*, 2025.