Morse Decomposition for Semi-Dynamical Systems with an Application to Systems of State-Dependent Delay Differential Equations

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Abstract

Understanding the structure of the global attractor is crucial in the field of dynamical systems, where Morse decompositions provide a powerful tool by partitioning the attractor into finitely many invariant Morse sets and gradient-like connecting orbits. Building on Mallet-Paret's pioneering use of discrete Lyapunov functions for constructing Morse decompositions in delay differential equations, similar approaches have been extended to various delay systems, also including state-dependent delays. In this talk, we develop a unified framework assuming the existence and some properties of a discrete Lyapunov function for a semi-dynamical system on an arbitrary metric space, and construct a Morse decomposition of the global attractor in this general setting. We demonstrate that our findings generalize previous results; moreover, we apply our theorem to a cyclic system of differential equations with threshold-type state-dependent delay.

References

[1] Balázs, I., Garab, Á., Rauscher, T.: Morse Decomposition for Semi-Dynamical Systems with an Application to Systems of State-Dependent Delay Differential Equations. Journal of Dynamics and Differential Equations (2025)