

On Parameter-Dependent Inhomogeneous Boundary-Value Problems in Sobolev Spaces

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Abstract

We study a wide class of linear inhomogeneous boundary-value problems for r th order ODE-systems depending on a parameter μ belonging to a general metric space. The solutions belong to the Sobolev spaces $(W_p^{n+r})^m$, $n \in \mathbb{N} \cup \{0\}$, $m, r \in \mathbb{N}$, $1 \leq p \leq \infty$. The boundary conditions are of a most general form $By = c$, where B is an arbitrary continuous operator from $(W_p^{n+r})^m$ to $\mathbb{C}^{rm}$. Thus, they may contain derivatives of the unknown vector function of integer and/or fractional orders $\geq r$. We find necessary and sufficient conditions for the continuity of solutions with respect to the parameter μ . We also prove that the solutions of the original problems can be approximated in the space $(W_p^{n+r})^m$ by solutions of ODE-systems with polynomial coefficients, right-hand sides of the equation, and multipoint boundary conditions, which are independent of the right-hand sides of the original problem.

References

- [1] O. Atlasiuk, V. Mikhailets, J. Taskinen, Parameter-dependent inhomogeneous boundary-value problems in Sobolev spaces, Preprint (2025) <https://doi.org/10.48550/arXiv.2512.21361>