Evolutionary Equations with State-Dependent Delay

Bernhard Aigner

TU Bergakademie Freiberg, Institute for Applied Analysis, Germany
BERNHARD.AIGNER@DOKTORAND.TU-FREIBERG.DE

Abstract

In [3] well-posedness in $\mathrm{H}^1(0,T;H)$ of state-dependent ODEs of the form

$$\begin{aligned} \dot{x}(t) &= F(t, x_{(t)}) \quad t \in [0,T] \\ x_{(0)} &= \Phi \end{aligned}$$

was established for Lipschitz-continuous prehistories $\Phi \in \mathrm{H}^1(-h,0;H)$ (where H is some Hilbert space) and almost uniformly Lipschitz-continuous forcing terms $F\colon [0,T] \times \mathrm{L}^2(-h,0;H) \to H$. The proof utilizes a Hilbert space projection argument and exponentially weighted Bochner–Lebesgue-spaces to force contractivity of a fixed point map. This approach can be generalized to evolutionary partial differential equations in the sense of R. Picard, that is, to equations of the form

$$\big(\partial_t M(\partial_t) + A\big)u(t) = f(t) \quad t \in \mathbb{R},$$

where $A\colon H \supseteq \mathrm{dom}(A) \to H$ is an m-accretive (unbounded) linear operator and $M\colon \mathbb{C}_{\mathrm{Re}>\nu} \to \mathcal{L}_{\mathrm{b}}(H)$ is a material law. This regime is general enough to capture a vide variety of examples, including classical PDEs (heat, wave and Maxwell's equations), examples from semigroup theory, port-Hamiltonian systems, as well as equations featuring fractional derivatives and convolutions (in time) with bounded operators.
The corresponding generalized initial value problem — that stems from a distributional formulation — is

$$\big(\partial_t M(\partial_t) + A\big)u(t) = f(t) + F\big(t, u_{(t)}\big) \quad t \in \mathbb{R}.$$

Local well-posedness (in the sense of weak solutions) of this equation similarly requires prehistories in H^1 with bounded derivative and a Lipschitz-property, but technical reasons additionally mandate a regularity increasing right-hand side and a consistency condition.

References

[1] *B. Aigner* and *M. Waurick*, “Evolutionary equations with state-dependent delay”, Preprint, arXiv:2511.14883 (2025) — accepted for publication in Nonlinear Anal.

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[3] *J. Frohberg* and *M. Waurick*, “State-dependent Delay Differential Equations on H^1”, J. Differ. Equations 410, 737–771 (2024)