

Potential and flux reconstructions for optimal a priori and a posteriori error estimates

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Abstract

Given a scalar-valued discontinuous piecewise polynomial, a **potential reconstruction** is a piecewise polynomial that is **trace continuous**, i.e., H^1 -conforming. It is best obtained via a conforming finite element solution of **local** homogeneous **Dirichlet problems** on vertex patches, using the finite element method. Similarly, given a vector-valued discontinuous piecewise polynomial not having the target divergence, a **flux reconstruction** is a piecewise polynomial that is **normal-trace continuous**, i.e., $\mathbf{H}(\text{div})$ -conforming, and has the target divergence. It is best obtained via **local** homogeneous **Neumann problems**, using the mixed finite element method.

The concepts of potential and flux reconstructions are known to lead to guaranteed, locally efficient, and polynomial-degree-robust **a posteriori error estimates**. Such use is based on the Prager–Synge equality and piecewise polynomial extension operators that we recall here.

We show that potential and flux reconstructions can also be used to obtain novel results in a priori error analysis. They actually allow to devise **stable local commuting projectors** that lead to p -robust **equivalence of global-best approximation** over the whole computational domain by a continuous (and constrained) finite element space with **local-best approximations** over individual mesh elements without any continuity requirement along mesh faces (and without any divergence constraint). Therefrom, optimal hp **approximation/a priori error estimates** under minimal elementwise Sobolev regularity follow. The main additional tools include here p -stable local decompositions.

References

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