On the dispersive effect of internal gravity waves in two-phase incompressible viscous flows

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Abstract

In this talk, we consider the dispersive effect of internal gravity waves in two-phase incompressible viscous flows in the N-dimensional Euclidean space $\mathbb{R}^N$ for $N \geq 2$. The two fluids are separated by a horizontal interface, and surface tension is taken into account at the interface. We call the fluids above and below the interface the upper fluid and lower fluid, respectively. If the upper fluid is heavier than the lower fluid, the well-known Rayleigh–Taylor instability occurs. In this study, on the other hand, we focus on the stable case where the upper fluid is lighter than the lower fluid. Under this stable setting, the boundary symbol associated with the linearized problem admits two zeros $\lambda_\pm$ satisfying $\lambda_\pm = \pm ic_1|\xi'|^{1/2} - c_2(1 \pm i)|\xi'|^{5/4} + o(|\xi'|^{5/4})$ as $|\xi'| \to 0$, where $i = \sqrt{-1}$ and $\xi' \in \mathbb{R}^{N-1}$ is the Fourier transform variable corresponding to the tangential direction $x' = (x_1, \ldots, x_{N-1})$. Here, c_1 and c_2 are positive constants depending on the densities, the viscosity coefficients, and the acceleration of gravity. The leading terms $\pm ic_1|\xi'|^{1/2}$ represent dispersive waves known as internal gravity waves. By utilizing both the dispersive effect of these waves and the fractional dissipation $-c_2|\xi'|^{5/4}$, we establish L_p-L_q decay estimates for the two-phase Stokes semigroup. If time permits, we shall discuss applications of these estimates to the nonlinear problem. This is based on joint work with Xin Zhang (Tongji University).