Multiharmonic algorithms for contrast-enhanced ultrasound

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Abstract

Nonlinear acoustic wave models are increasingly important for medical ultrasound and industrial applications where sound propagates at high intensities or frequencies. Harmonic generation, in particular, plays a key role in contrast-enhanced ultrasound, both for imaging and therapeutic applications. In this talk, we will discuss wave–microbubble models, where the acoustic field is governed by a nonlinear Westervelt-type wave equation coupled to a Rayleigh-Plesset-type ODE describing the bubble dynamics. This includes the analytical and computational questions arising from this coupling, such as the existence of time-periodic solutions and handling nonlinearity and differing time scales. Since time-domain approaches are computationally demanding for capturing nonlinear effects, we will introduce an alternative strategy based on a multiharmonic Ansatz applied to the coupled wave–bubble system. A priori error estimates will be presented that characterize the approximation error in terms of the number of retained harmonics and a contribution arising from the fixed-point iteration. Additionally, numerical experiments will illustrate how the number of retained harmonics and the presence of microbubbles influence acoustic wave propagation.

The talk is based on joint works [1, 2].

References

[1] V. Nikolić and T. Rauscher, *Mathematical models for nonlinear ultrasound contrast imaging with microbubbles*, SIAM Journal on Applied Mathematics, 85 (2025), pp. 961–982.

[2] V. Nikolić and T. Rauscher, *Multiharmonic algorithms for contrast-enhanced ultrasound*, Journal of Scientific Computing, (to appear).