Global solvability of the Q-tensor model for nematic liquid crystals

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Abstract

The molecules of nematic liquid crystal flow as in a liquid phase; however, they have an orientation order. The orientation order is described by the symmetric, traceless matrix **Q**. Beris and Edwards proposed a Q-tensor model, the coupled system of the Navier-Stokes equations and a parabolic-type equation describing the evolution of the order parameter **Q**, to represent nematic liquid crystal flows. The aim of this talk is to prove the unique existence of the global-in-time solution in the maximal regularity class for the Q-tensor model in the half-space $\mathbb{R}^N_+$, $N \geq 2$. Concerning the existence results for the strong solutions in the maximal regularity class, [2] proved the local well-posedness in $\mathbb{R}^N_+$ for small initial data. The global well-posedness was investigated in $\mathbb{R}^N$ by [4, 5], while that in the bounded domain was studied by [3, 6].

In this talk, we especially discuss the weighted estimates of solutions for the linearized problem in the L_p-in-time and L_q-in-space setting to obtain the global solvability in $\mathbb{R}^N_+$ for small initial data. The key issue is to estimate the lower order terms by the semigroup theory. This talk is based on [1].

References

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