The strong unstable manifold and periodic solutions in differential delay equations with cyclic monotone negative feedback

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Abstract

For (N+1)-dimensional systems of differential delay equations with cyclic and monotone negative feedback, we construct a two-dimensional invariant manifold, on which phase curves spiral outward towards a bounding periodic orbit. We assume essentially only instability of the zero equilibrium. Methods of the Poincaré-Bendixson theory due to Mallet-Paret and Sell are combined with techniques used by Walther for the scalar case ($N = 0$). Statements on the attractor location and on parameter borders concerning stability and oscillation are included. The results apply to models for gene regulatory systems, e.g. the 'repressilator' system.

References

[1] Ivanov, A. F., and Lani-Wayda, B.: *The strong unstable manifold and periodic solutions in differential delay equations with cyclic monotone negative feedback.* Commun. Pure Appl. Anal. (CPAA) Vol. **24**, Iss. 9 (2025), 1713-1752. Doi: 10.3934/cpaa.2025057