New Banach spaces-based mixed finite element methods for steady-state flows of magnetic fluids

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Abstract

In this paper we introduce and analyze new mixed finite element methods for solving the steady-state flows of magnetic fluids. To this end, we consider two different ways of deriving the stationary model from the time-dependent problem, namely, either by dropping the time derivatives or by replacing them by a zero order term. In this way, and motivated by our interest in mixed approaches, we are led to systems of equations for the fluid having as unknowns, in the first case, the velocity, its gradient, and a partial stress magnetic tensor, which is given by the sum of the Maxwell and hydrodynamics stress tensors, and the half of the convective term. Similarly, in the second one they turn out to be the strain rate tensor, the velocity, the full stress magnetic tensor, and the vorticity tensor. Next, we introduce a potential unknown so that the magnetic field satisfies a Neumann boundary value problem with the normal component of the applied magnetic field as the corresponding boundary condition. The resulting mixed variational formulations for the fluid fit the structures of a nonlinear saddle-point problem, and a nonlinear perturbation of, in turn, a perturbed twofold saddle-point problem, respectively, whereas the one for the magnetic field is given by a usual saddle-point setting. Hence, fixed-point strategies, along with the generalized Babuška-Brezzi theory in Banach spaces and recent related abstract results, are employed to establish the well-posedness of the associated continuous and discrete schemes. Finally, several numerical results illustrating the performance of our methods are reported. [`This is a joint work with G.A. Benavides, S. Caucao and Y.D. Sobral`].

References

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