Landmarks in the History of Linear and Nonlinear Preconditioning

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Abstract

Preconditioning of linear systems remains a highly active area of research, and nonlinear preconditioning has recently emerged as an important new direction. Over the past decade, a unifying principle has become apparent in both the linear and nonlinear settings: effective preconditioners are best developed first as standalone iterative methods, rather than being designed directly as auxiliary components for Krylov or Newton-type methods. Once a robust and well-understood iterative method has been constructed, Krylov methods or Newton's method can be viewed as systematic acceleration mechanisms that enhance performance and improve robustness.

This perspective is strongly reflected in the historical development of iterative methods. I will review key landmarks in the evolution of iterative techniques for both nonlinear and linear problems. The earliest known iterative schemes were developed for nonlinear problems—most notably the Babylonian method for computing square roots—which later evolved into general fixed-point iterations. A major conceptual breakthrough followed with the introduction of Newton's method.

Iterative methods for linear systems trace back to a letter from Gauss to Gerling, which laid the groundwork for stationary schemes such as Jacobi, Gauss–Seidel, and SOR. More powerful physics-based developments include multigrid and domain decomposition methods. In parallel, nonstationary iterations emerged, including extrapolation techniques and, closely related to them, Krylov subspace methods.

In both the linear and nonlinear settings, one observes a natural hierarchy: basic iterative solvers are complemented by systematic acceleration mechanisms—Newton's method in the nonlinear case and Krylov methods in the linear case. I will argue that viewing these accelerators as tools to enhance carefully designed iterative methods provides a historically grounded and conceptually effective framework for advancing research in linear and nonlinear preconditioning.

References

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