Exploring the relativistic Kepler problem: A journey through bounded orbits and bifurcation phenomena

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Abstract

The motion of a relativistic particle in a Kepler potential can be described by the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3}, \quad x \in \mathbb{R}^3 \setminus \{0\},$$

where $m > 0$ represents the mass of the particle, c is the speed of light, and $\alpha > 0$ is a constant. Firstly, we illustrate the Hamiltonian formulation of the problem and we focus our attention on the description of the periodic and quasi-periodic solutions. Secondly, we deal with the perturbed equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3} + \varepsilon\,\nabla_x U(t,x), \quad x \in \mathbb{R}^3 \setminus \{0\},$$

where $U(t,x)$ is T-periodic in the first variable and $\varepsilon \in \mathbb{R}$. We present different kinds of results concerning the existence of periodic orbits obtained in collaboration with Alberto Boscaggin (University of Torino), Walter Dambrosio (University of Torino), and Duccio Papini (University of Modena and Reggio Emilia).