Local Minimizers and Mountain Pass Critical Points

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Abstract

In this talk we highlight that the Mountain Pass Theorem should be viewed not only as an existence result but, more fundamentally, as a multiplicity theorem for critical points. A crucial role is played by the presence of a local minimizer: the well–barrier structure generated by a local minimum is precisely what allows the variational framework to produce an additional critical point of saddle type through a minimax argument. We then present an alternative proof of the limiting case of the Mountain Pass Theorem, developed without relying on advanced analytical tools. The proof is based on a new mountain pass lemma, obtained by means of an ε-perturbation technique in the spirit of Brezis–Nirenberg. Applications to differential problems naturally yield the existence of two or three solutions, depending on the growth of the nonlinearity—whether it is superlinear or sublinear. This perspective illustrates how the interplay between a local minimum and a mountain-pass configuration provides a simple yet powerful mechanism for obtaining multiplicity of solutions.

References

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